Quantum Signature of Chaos in SQUIDs

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近年の微細加工技術の進展によって、微小な超伝導量子干渉デバイス (SQUID) が作成可能となってきている。この系において、スペクトル統計を通して量子系におけるカオスの兆候を議論した。この系は 2 次元の周期ポテンシャル系と同等であり、そこでのバンド構造にスペクトル統計が現れることが特徴である。外部磁場などの外部パラメータを制御することにより、GUE 統計や GOE 統計で記述されるスペクトル統計が得られることを示す。

Recent develepment in fabrication techniques has enabled us to controll quantum states of mesoscopic systems. For application to quantum computation, much attention has been paid to fabricate a quantum bit (qubit) in these systems [1]. As a candidate for qubits, superconducting quantum interference devices (SQUIDs) have been studied intensively for several years. In order to construct a qubit in SQUIDs, only the lowest two quantum levels are focused on, while the other energy levels are neglected. In this paper, we consider quantum signature of chaos in SQUIDs by discussing spectral properties in quantum levels with higher energy which has not been studied.

We consider a SQUID model with three Josephson junctions as shown in Fig. 1 (a). External voltages, V_1 and V_2 are appled through capacitances to two of three superconducting islands separeted by the junctions. We assume that the inductunce of this SQUID is small. Then, the phase variable of the junction 3 can be determined by the variables of the other junctions as $\phi_3 = \phi_1 - \phi_2 - 2\pi f$, where f is a normalized magnetic flux through the SQUID. As a result, the Josephson energy is given as

$$H_{\rm J} = -E_{\rm J}\cos\phi_1 - E_{\rm J}\cos\phi_2 - E_{\rm J}\cos(\phi_1 - \phi_2 - 2\pi f). \tag{1}$$

The charging energy is given by the charge variables, n_i's conjugate to the phase variables as

$$H_{\rm C} = 4E_{\rm C}(n_1 - n_1^*)^2 + 4E_{\rm C}(n_2 - n_2^*)^2, \tag{2}$$

where n_1^* and n_2^* are normalized external voltages and $E_{\rm C} = e^2/2C$. If we regard ϕ_i and n_i as a position and a momentum respectively, the total Hamiltonian $H = H_{\rm C} + H_{\rm J}$ describes a particle in the two-dimensional periodic potential [2]. Generally, classical dynamics in SQUIDs show chaotic behaviors in a proper energy region [3]. Hence, we can expect quantum signature of chaos through spectral properties [4].

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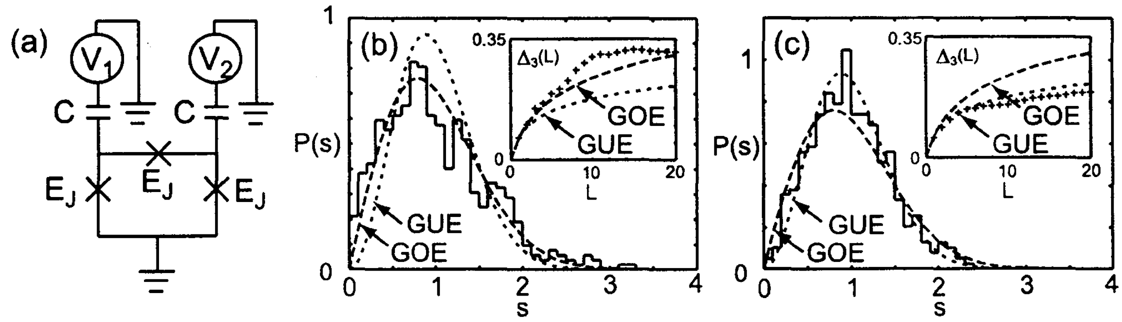

図 1: (a) A SQUID model considered in this paper. (b) Numerical results of level spacing distribution and spectral rigidity (inset) for $E_J = 1$, $E_C = 0.003$, $n_1^* = 0.25$, $n_2^* = 0$ and $f = 0.5$. (c) Results for $f = 0.25$.

Quantization of this system gives rise to band structures, and quantum levels are obtained for a fixed Bloch wave number. It should be noted that by combining the boundary condition with the gauge transformation, the Bloch number is determined by (n_1^*, n_2^*) [3]. The Josephson energy is rewritten in terms of annihilation(creation) operators of the charges at the junctions as

$$H_J = -\frac{E_J}{2}(b_1 + b_1^\dagger) - \frac{E_J}{2}(b_2 + b_2^\dagger) - \frac{E_J}{2}(b_1 b_2^\dagger e^{-2\pi i f} + b_1^\dagger b_2 e^{2\pi i f}). \tag{3}$$

From eq. (3), we can see that all the matrix elements are real only when f is an integer or a half integer. This implies that the spectral properties are described by the GOE statistics in this case. The results for $f = 0.5$ is shown in Fig. 1 (b) by using the levels in a energy range where chaotic behavior is dominant in classical dynamics. These results show that the level statistics belongs to GOE. On the other hand, the matrix elements become complex when f is not an integer nor a half integer. The spectral properties for $f = 0.25$ are shown in Fig. 1 (c). These results show that the level statistics belongs to GUE. Thus, in the present system, we can change the universality class by changing an external magnetic flux.

In order to observe this quantum signature of chaos through spectral properties, we have to perform level spectroscopy in SQUIDs in the condition $k_B T < E_C \ll E_J$ where T is a temperature. I hope that these results will be clarified by experiments in near future.

References

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